RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

M.A./M.Sc. THIRD SEMESTER EXAMINATION, JANUARY 2015

SECOND YEAR

MATHEMATICS

Date : 19/01/2015

Time : 11 am – 1 pm

Paper : XIV [Commutative Algebra]

Full Marks : 50

Answer as many questions as you can

Maximum marks is 50

1. State and prove prime avoidance lemma. Is it true for infinite case of prime ideals? [6]
2. Describe spec $\mathbb{R}[x]$, where $\mathbb{R}$ is the field of real numbers. [6]
3. Let M be an A module and I an ideal of A. Suppose that $M_m = 0$ for all maximal ideals $m \supset I$. Prove that M = IM. [6]
4. Let A be a ring in which every prime ideal is finitely generated. Show that A is Noetherian. [6]
5. Let A be a ring. Suppose that, for each prime ideal p, the local ring A_p has no nilpotent element $\neq 0$. Show that A has no nilpotent element $\neq 0$. If each A_P is an integral domain, is A necessarily an integral domain? [4+2]
6. Show that in the rind A[x], the Jacobson radical is equal to nilradical. [6]
7. In the ring k[x, y, z] (k is a field), find the primary decomposition of the ideal (x^2, xy, xz, yz). Which components are isolated and which are embedded? [6]
8. Let $A \rightarrow B$ be a ring homomorphism and let p be a prime ideal of A. Show that p is the contraction of a prime ideal of B if and only if $p^{ec} = p$. [6]
9. Show that in an Artin ring every prime ideal is maximal. [6]
10. Let A be a commutative ring with identity. Show that $A = A_1 \times A_2$ iff there is an idempotent element (i.e. $e^2 = e$) in A. [6]

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